12121950502 4

Code No.1277/CBCS/N

FACULTY OF SCIENCE
M.Sc. II – Semester Examination, December 2020

Subject: Mathematics
Paper – IV Topology

Time: 2 Hours **Max.Marks: 80**

PART – A

Note: Answer any five questions. **(5x7 = 35 Marks)**

1 If T_1 and T_2 are two topologies on a set X, then show that $T_1 \cap T_2$ is also a Topology on X.
2 Let (X,T) be a Topological space and A,B are subsets of X then show that $\overline{A \cup B} = \overline{A} \cup \overline{B}$
3 Show that any closed subspace of a compact space is compact.
4 Show that every totally bounded metric space is bounded
5 Define T_1-space and show that any subspace of a T_1-space is again a T_1-space.
6 Define a normal space and give an example.
7 If ϕ and X are the only sets which are both open and closed in a topological space X, then show that X is connected.
8 Show that any continuous image of a connected space is connected.

PART – B

Note: Answer any three questions. **(3x15 =45 Marks)**

9 State and prove Lindelof's theorem.

10 (i) Let (X,T) be a Topological space and A is a closed subset of X then show that A is the disjoint union of the set of all isolated points of A and the set of all limit points of A.
(ii) Let (X,T) be any Topological space and A is any subset of X then show that $(\overline{A})^c = (A^c)^0$ where A^c denotes the complement of A.

11 (i) Define Basic open cover, sub basic open cover for a Topological space.
(ii) Show that a Topological space is compact if and only if every basic open cover has a finite sub cover.

12 State and prove Lebesgue covering Lemma.

13 Define a complete regular space. Also show that a complete regular space is a Haussdorff space .

14 State and prove Urysohn's Lemma.

15 Show that a topological space X is disconnected if and only if there exists a continuous mapping of X onto discrete two point space {0, 1}.

16 Let X be any topological space then
i) Define component of X.
ii) Show that each point of X is contained in exactly one component of X.
iii) Show that each connected subspace of X is contained in a component of X.

*****

121219505024

Code No. 1279/CORE

FACULTY OF SCIENCE

M.Sc. II-Semester Examinations, December 2020

Subject: Mathematics / Applied Mathematics

Paper : V – Theory of Ordinary Differential Equations

Time: 2 Hours | **Max. Marks: 80**

PART – A

Answer any five questions. **(5x7=35 Marks)**

1 Prove that x^4 and $|x|x^3$ are linearly independent functions on [-1, 1] but they are linearly dependent on [-1, 0] and [0, 1].

2 Prove that there are three linearly independent solutions of the third order equation $x''' + b_1(t)x'' + b_2(t)x' + b_3(t)x = 0,\ t \in I$ where b_1, b_2, and b_3 are functions defined and continuous on an interval I.

3 Prove that the error $x(t) - x_n(t)$ satisfies the estimate $|x(t) - x_n(t)| \le \frac{L(Kh)^{n+1}}{K(n+1)!}e^{kh}$ where $t \in [t_o, t_o + h]$.

4 Find the largest interval of existence of the solution of the IVP $x' = x^2 + \cos^2 t,\ x(0) = 0$ where R is the rectangle containing (0, 0) and $R = \{(t, x) \mid 0 \le t \le a, |x| \le b, a \ge \frac{1}{2}, b > 0\}$.

5 Define:
(i) Maximal solution (ii) Minimal solution of the IVP $x' = f(t, x), x(t_o) = x_o$ on $[t_o, t_o + h]$.

6 Suppose that $f(t, x)$ is non-increasing in x. Then show that
(i) there exist lower and upper solutions v_o, w_o of $x' = f(t, x), x(t_o) = x_o$ such that $v_o \le w_o$ on $I = [t_o, t_o + h]$.
(ii) there exists a unique solution x of $x' = f(t, x), x(t_o) = x_o$ on I such that $v_o \le x \le w_o$.

7 Prove that the second order linear differential equation $L_2(y) = a_o(x)y'' + a_1(x)y' + a_2(x)y = 0$ is self adjoint if and only if $a_0'(x) = a_1(x)$.

8 Let u and v be any two solutions of self adjoint equation of order two of the form $(r(x)y')' + p(x)y = 0$ where $r(x) \neq 0, r'$ and p(x) are continuous functions on [a, b]. Then show that $r(x)[u(x)v'(x) - u'(x)v(x)] = K$ where K is a constant.

p..2

PART – B

Answer any three questions. **(3x15=45 Marks)**

9 Let the functions $b_1, b_2, ..., b_n$ in $L(x) = x^{(n)} + b_1(t)x^{(n-1)} + ... + b_n(t)x = 0, t \in I$ be defined and continuous on an interval I. Let $\varphi_1, \varphi_2, ..., \varphi_n$ be n linearly independent solutions of $L(x) = 0$ existing on I containing a point t_o. Then prove that $w(t) = \exp\left[-\int_{t_o}^{t} b_1(s)ds\right] w(t_o), \quad t_o, t \in I$.

10 (i) Solve $x'' + x' = 4t^2 e^t$.
(ii) Solve $x'' - 4x' = t\, e^{4t}$.

11 Let $x(t) = x(t, t_o, x)$ and $x^*(t) = x(t, t_o^*, x_o^*)$ be solutions of the IVPs $x' = f(t, x), x(t_o) = x_o$ and $x' = f(t, x), x(t_o^*) = x_o^*$ respectively on an interval $a \le t \le b$. Let $(t, x(t)), (t, x^*(t))$ lie in a domain D for $a \le t \le b$. Further, let $f \in$Lip(D,K) be bounded by L in D. Then show that for any $\in > 0$, there exists a $\delta = \delta(\in) > 0$ such that $|x(t) - x^*(t)| < \in, a \le t \le b$ whenever $|t_o - t_o^*| < \delta$ and $|x_o - x_o^*| < \delta$.

12 Assume that $f(t, x)$ is continuous on the strip S defined by $S : |t - t_o| \le T$ and $|x| \le \infty$ where T is some finite positive real number. Let $f \in Lip(S, K)$. Then prove that the successive approximations defined by $x_n(t) = x_o + \int_{t_o}^{t} f(s, x_{n-1}(s))ds, n = 1,2,3,....$ for the IVP $x' = f(t, x), x(t_o) = x_o$ exist on $|t - t_o| \le T$ and converge to a solution x of $x' = f(t, x), x(t_o) = x_o$.

13 Let $v, w \in C'\{[t_o, t_o + h), R\}$ be lower and upper solutions of $x' = f(t, x), x(t_o) = x_o$ respectively. Suppose that, for $x \ge y$, f satisfies the inequality $f(t, x) - f(t, y) \le L(x - y)$ where L is a positive constant. Then prove that $v(t_o) \le w(t_o)$ implies that $v(t) \le w(t), t \in (t_o, t_o + h)$.

14 Let $f \in C\{[t_o, t_o + h) \times R, R\}$ and $|f(t, x)| \le M$. Then prove that there exists a solution of the IVP $x' = f(t, x), x(t_o) = x_o$ on $[t_o, t_o + h)$.

15 Let $u, v \in C'[\alpha, \beta]$. Let $v(\alpha) = v(\beta) = 0$ and $v(x) \ne 0, x \in (\alpha, \beta)$. If $w(u, v)(x) = u(x)v'(x) - u'(x)v(x) \ne 0$ on [α, β], then prove that v vanishes precisely once in the interval (α, β) and also zero of u is simple.

16 State and prove Sturm comparison theorem.

*****